Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

3. If a steel wire 2 m in length weighs 0.9 nt (about 0.20 lb) and is stretched by a tensile force of 300 nt (about 67.4 lb), what is the corresponding speed of transverse waves?

The formula for the speed of transverse waves is just the sqrt of the tension divided by the density.

$$\text{density} = \frac{.9}{(2*9.8)} \quad (*\ \text{nts}/(\text{m}/\text{sec}^2)\ *)$$

```
0.0459184
```

```
tensile = 300 (* nts *)
```

```
300
```

$$\text{spd} = \sqrt{\frac{\text{tensile}}{\text{density}}}$$

```
80.829
```

```
80.82903768654761`    (* sqrt(m/sec^2) *)
```

```
80.829
```

Obtaining text answer. Source: *https://www3.nd.edu/~apaul2/The_Jungle/PHYS31210L_-files/E%2010-Standing%20Waves.pdf*

Having some trouble at first with the next group of problems, I decided to go through example 1 on p. 550. First I tried to make everything adjustable, but I could not get it to work.

```
In[32]:= ClearAll["Global`*"]

c = 1; L = 1; k = 0.01;
f[x_] = k Sin[π x]
0.01 Sin[π x]

A[n_] =
  (2 / L) Integrate[f[x] * Sin[n π x / L], {x, 0, L}] // Assumptions → n > 1
```

$$(\text{Assumptions} \to n > 1)\left[-\frac{0.0063662\ \text{Sin}[n\,\pi]}{-1. + n^2}\right]$$

```
Lambda[n_] = (c n π / L) ^2
```

$$n^2\,\pi^2$$

```
u[x_, t_, N_] := Sum[A[n] Cos[Sqrt[Lambda[n]] t] Sin[n π x / L], {n, 2, N}]
```

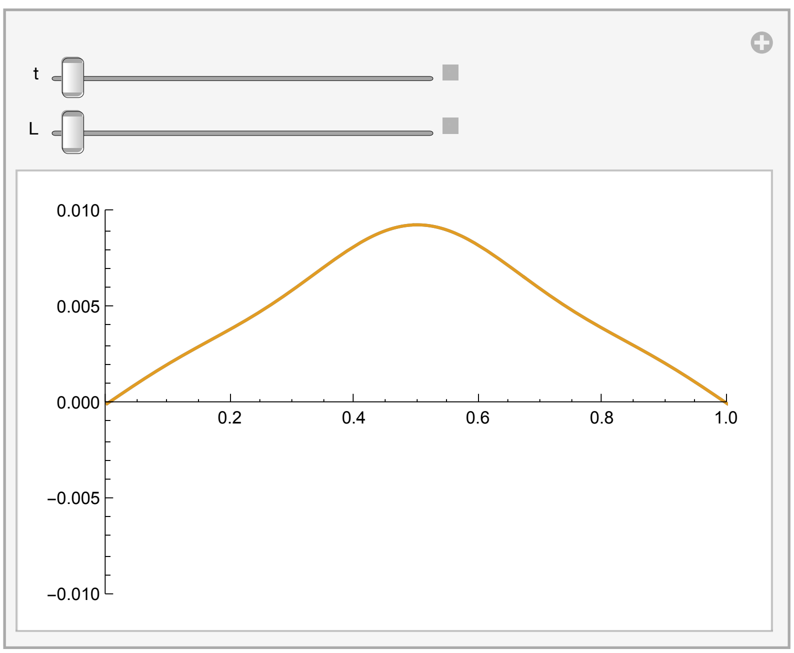
```
u4[x_] := Sum[A[n] Cos[Sqrt[Lambda[n]]] Sin[n π x / 1], {n, 2, 4}]

u[x, 1, 4]

Sin[2 π x] (Assumptions → True) [0.] -
 Sin[3 π x] (Assumptions → True) [0.] + Sin[4 π x] (Assumptions → True) [0.]
```

Then I felt forced to simpify things, and reduced the series solution to example 1, at the bottom of p. 550, to a very simplified three terms and wrote the **Manipulate** below. It works, though the triangularity is not as sharply defined as in figure 291. If the manipulate window is dropped down to expose the input fields, it will be seen that, within the intervals of L and t defined in the **Manipulate,** it approximates figure 291 for each of the partial t and L inputs shown there.

```
Manipulate[
 Plot[{8 (0.01)/π^2 (Sin[π (x - L)] Cos[π t] - 1/9 Sin[3 π (x - L)] Cos[3 π t] +
      1/25 Sin[5 π (x - L)] Cos[5 π t]), 8 (0.01)/π^2 (Sin[π (x + L)] Cos[π t] -
      1/9 Sin[3 π (x + L)] Cos[3 π t] + 1/25 Sin[5 π (x + L)] Cos[5 π t])},
  {x, 0, 1}, PlotRange → {{0, 1}, {-0.01, .01}}], {t, 0, 1}, {L, 0, 1}]
```



5 - 8 Graphing Solutions

Using numbered line (13), p. 555, sketch or graph a figure (similar to Fig. 291 in Sec. 12.3) of the deflection u(x,t) of a vibrating string (length L=1, ends fixed, c=1) starting with initial velocity 0 and initial deflection (k small, say, k=0.01).

5. f(x)=k sin πx

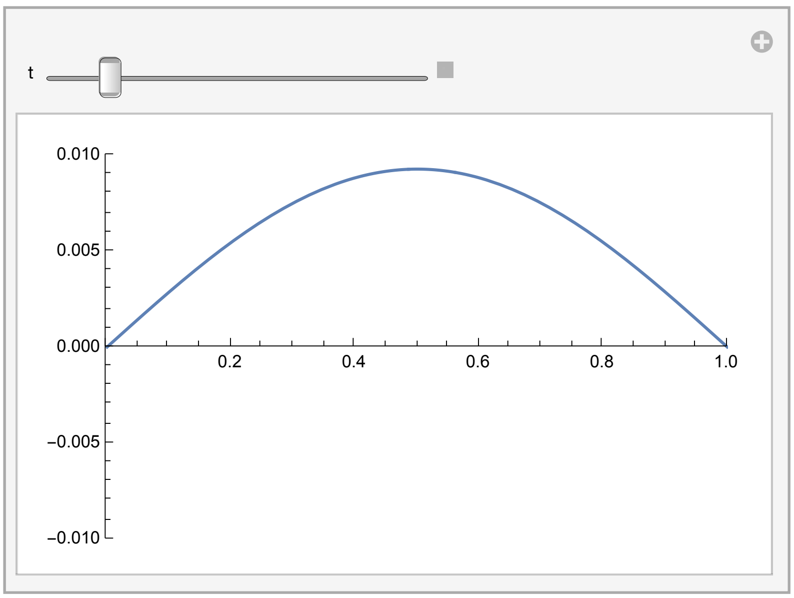
In[33]:=
```
ClearAll["Global`*"]
```

The problem description advises use of numbered line (13) on p. 555. Interpreting that, I would find

$$u(x, t) = \frac{k}{2}[\operatorname{Sin}[\pi (x + c t)] + \operatorname{Sin}[\pi (x - c t)]]$$

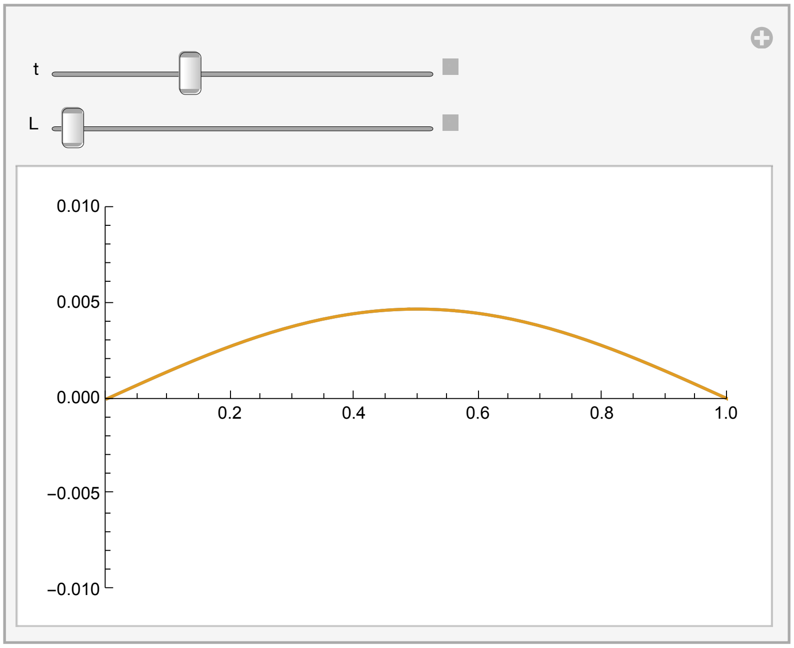
With the values of c and k in the problem description, I think this might be equivalent to

```
Manipulate[Plot[{1/2 (0.01 Sin[π (x + t)] + 0.01 Sin[π (x - t)])},
  {x, 0, 1}, PlotRange → {{0, 1}, {-0.01, .01}}], {t, 0, 1}]
```



The above plot does not do very much. There is no help from the answer appendix, but I believe the plot needs to be modeled after the one in example 1 on p. 550. That would make it like the following, resembling figure 291.

```
Manipulate[Plot[{1/2 (0.01 Sin[π (x + t) - L] + 0.01 Sin[π (x - t) - L]),
    1/2 (0.01 Sin[π (x + t) + L] + 0.01 Sin[π (x - t) + L])}, {x, 0, 1},
   PlotRange → {{0, 1}, {-0.01, .01}}], {t, 0, 1}, {L, 0, π}]
```

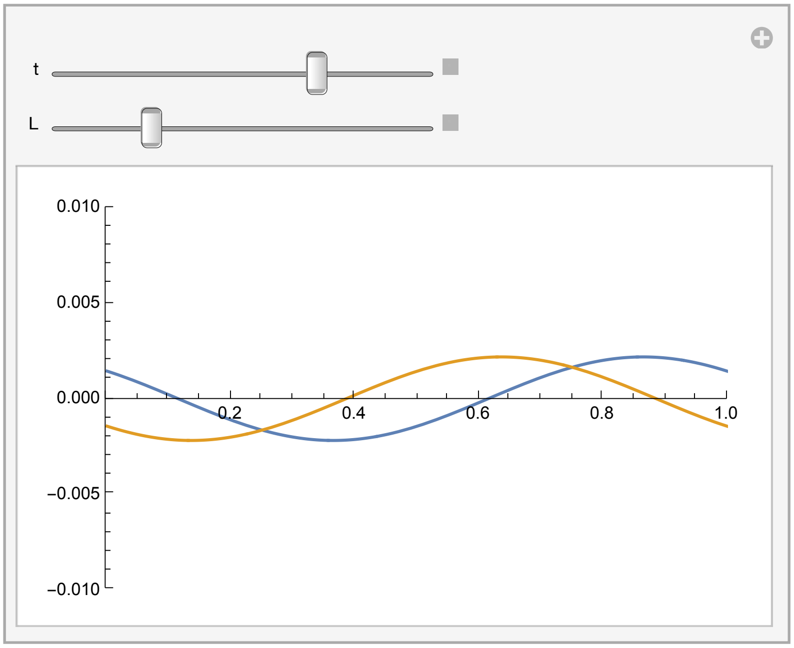


7. f(x)=k sin 2πx

This problem is very much like the last. Taking the same approach, and without any input from the answer appendix, I come up with

```
In[34]:= ClearAll["Global`*"]
```

```
Manipulate[Plot[{1/2 (0.01 Sin[2 π (x + t) - L] + 0.01 Sin[2 π (x - t) - L]),
   1/2 (0.01 Sin[2 π (x + t) + L] + 0.01 Sin[2 π (x - t) + L])}, {x, 0, 1},
  PlotRange → {{0, 1}, {-0.01, .01}}], {t, 0, 1}, {L, 0, π}]
```



The 2π instead of π in this problem makes the plot show additional activity, but little if any additional interest.

9 - 18 Normal Forms

Find the type, transform to normal form, and solve.

Finding the type means checking numbered line (14) and the table on p. 555.

9. $u_{xx} + 4\, u_{yy} = 0$

Elliptic. As far as normal form is concerned, the form presented below is normal for **DSolve**.

```
In[35]:= ClearAll["Global`*"]

a = 1; b = 0; c = 4;
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
4 u^(0,2)[x, y] + u^(2,0)[x, y] == 0

sol = DSolve[eqn, u, {x, y}]

{{u → Function[{x, y}, C[1][2 i x + y] + C[2][-2 i x + y]]}}
```

The green cell above matches the text answer, with the understanding that C[1] and C[2] are considered functions by the text.

From this point to the end of this problem section, I deviate from the text and s.m. in procedure, because I could not catch on to the s.m. or text in terms of details. For source, I use a YouTube video, *https://www.youtube.com/watch?v=rO-f3zh3_kw,* time 0:00-5:12 and 7:21-11:15. The differences are as follows: (1) the coefficient of the u_{xy} factor is b, not b/2; (2) the middle coefficient of the characteristic equation has positive sign, and (3) the defining equations go like so:
Hyperbolic: u(x,y) = $f(m_1 x + y) + g(m_2 x + y)$ [yielding two unequal real roots]
Elliptic: u(x,y) = $f(m_1 x + y) + g(m_2 x + y)$ [yielding two complex roots]
Parabolic: u(x,y) = $f(m_1 x + y) + x\,g(m_2 x + y)$ [yielding two equal real roots]
(Note the extra x in Parabolic.)

11. $u_{xx} + 2\,u_{xy} + u_{yy} = 0$

```
In[36]:= ClearAll["Global`*"]

a = 1; b = 2; c = 1; b^2 - 4 a c
0
```

Parabolic.

For solns of form u(x,y) = f(mx +y) + g(mx +y) the characteristic equation is am^2 +bm + c = 0 = $m^2 + 2m + 1 = 0$.

```
vi = Factor[m^2 + 2 m + 1 == 0]
(1 + m)^2 == 0

Solve[vi, m]
{{m → -1}, {m → -1}}
```

Therefore the two functions sought will be:

```
funcf = (x - y)
x - y

funcg = x (x - y) (* with extra x added for parabolic*)
x (x - y)
```

The green cells above match the answer in the text.

13. $u_{xx} + 5\,u_{xy} + 4\,u_{yy} = 0$

Hyperbolic.

```
In[37]:= ClearAll["Global`*"]
```

```
a = 1; b = 5; c = 4; b^2 - 4 a c
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
9
```

$4\, u^{(0,2)}[x, y] + 5\, u^{(1,1)}[x, y] + u^{(2,0)}[x, y] == 0$

```
characteqn = m^2 + 5 m + 4 == 0
4 + 5 m + m^2 == 0
```

```
vi = Factor[characteqn]
(1 + m) (4 + m) == 0
```

```
Solve[vi, m]
{{m → -4}, {m → -1}}
```

Therefore the eqns sought will be:

```
f[x_, y_] = y - 4 x
g[x_, y_] = y - x
```

```
-4 x + y
```

```
-x + y
```

The green cells above match the answer in the text.

15. $xu_{xx} - yu_{xy} = 0$

Hyperbolic. This one does not match the template, because the coefficients are not constants.

```
In[17]:= ClearAll["Global`*"]
```

```
In[22]:= a = 1; b = -1; c = 0; b^2 - 4 a c
         eqn = x * D[u[x, y], {x, 2}] - y * D[u[x, y], x, y] == 0
Out[22]= 1
```

Out[23]= $-y\, u^{(1,1)}[x, y] + x\, u^{(2,0)}[x, y] == 0$

```
In[24]:= DSolve[{eqn}, u, {x, y}]
```

Out[24]= $\text{DSolve}\left[\left\{-y\, u^{(1,1)}[x, y] + x\, u^{(2,0)}[x, y] == 0\right\}, u, \{x, y\}\right]$

As shown in the cell above, Mathematica rejects the project. An appeal to Stack Exchange yielded a response from Michael E2 which seems to solve it. (*https://mathematica.stackexchange.com/questions/201887/dsolve-does-not-solve-2nd-order-linear-pde-with-variable-coefficients*). The details:

In[25]:=
```
Fold[Function[{eq, var}, DSolve[eq /. First[#1], var, {x, y}]] @@ #2 &,
 {{}}, Transpose@{{x * D[v[x, y], {x}] - y * D[v[x, y], y] == 0,
    D[u[x, y], x] == v[x, y]}, {v, u}}]
```

Out[25]= $\{\{u \to \text{Function}[\{x, y\}, \int_1^x C[1][y\,K[1]]\,dK[1] + C[2][y]]\}\}$

The above cell output looks funny at first. Both C[·] and K[·] are treated as functions. Evaluating the integral,

In[26]:= $\int_1^x y\,K\,dK$

Out[26]= $-\frac{y}{2} + \frac{x^2 y}{2}$

And adding an arbitrary contribution from C[2]

In[27]:= $u[x\_, y\_] = -\frac{y}{2} + \frac{x^2 y}{2} + y^2$

Out[27]= $-\frac{y}{2} + \frac{x^2 y}{2} + y^2$

And testing,

In[31]:=
```
x * D[u[x, y], {x, 2}] - y * D[u[x, y], x, y]
```

Out[31]= 0

It seems to work. The text answer, u = $\frac{1}{y} * f_1(x*y) + f_2(y)$, also works. I assume I will find out later if the technique can be modified for initial conditions.

17. $u_{xx} - 4\,u_{xy} + 5\,u_{yy} = 0$

Elliptic.

In[38]:=
```
ClearAll["Global`*"]
```

```
a = 1; b = -4; c = 5; b^2 - 4 a c
eqn = a * D[u[x, y], {x, 2}] + b * D[u[x, y], x, y] + c * D[u[x, y], {y, 2}] == 0
```

-4

$5\,u^{(0,2)}[x, y] - 4\,u^{(1,1)}[x, y] + u^{(2,0)}[x, y] == 0$

```
characteqn = m^2 - 4 m + 5 == 0
```

$5 - 4\,m + m^2 == 0$

```
vi = Factor[characteqn]
```

$5 - 4\,m + m^2 == 0$

```
Solve[vi, m]
```

$\{\{m \to 2 - i\}, \{m \to 2 + i\}\}$

Therefore the two sought functions will be:

```
f[x_, y_] = x (2 - i) + y
g[x_, y_] = x (2 + i) + y
```

```
(2 - i) x + y
```

```
(2 + i) x + y
```

The functions shown in the green cell above match the text answer (after digesting signs).